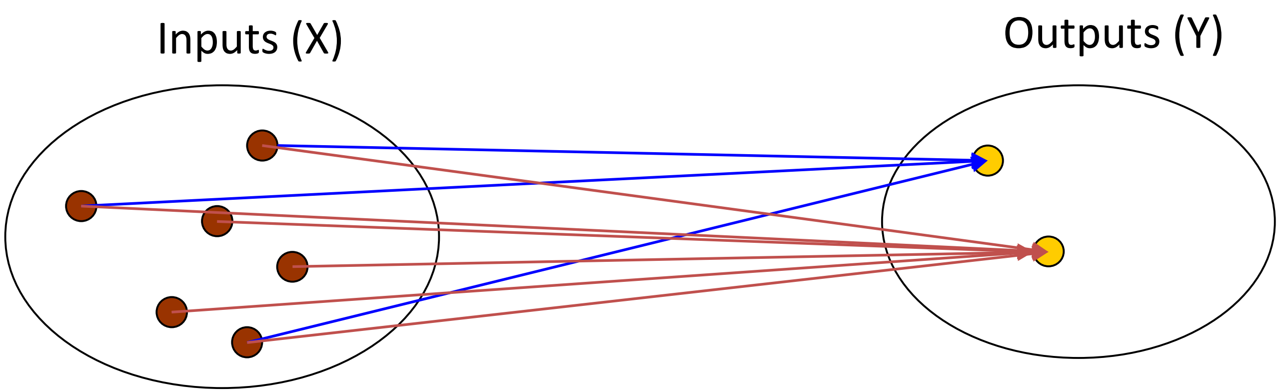
Costs and supply

Microeconomics

The firms technology: input and output

- We start by introducing the production function, which describes the firm's technology.
- An **input** (or **factor of production**) is a good or service used to produce output[s] (other goods or services).



$$\underline{y} = f(\underline{x}) \quad : \underline{x} \in X; \ \underline{y} \in Y; \quad f(\underline{x}_i) \geq \underline{y}_i \ \forall i$$

$\underline{y}$: output(s); $\underline{x}$: input(s); f: production function; Y: set of outputs; X: set of inputs

Inputs (factors of production)

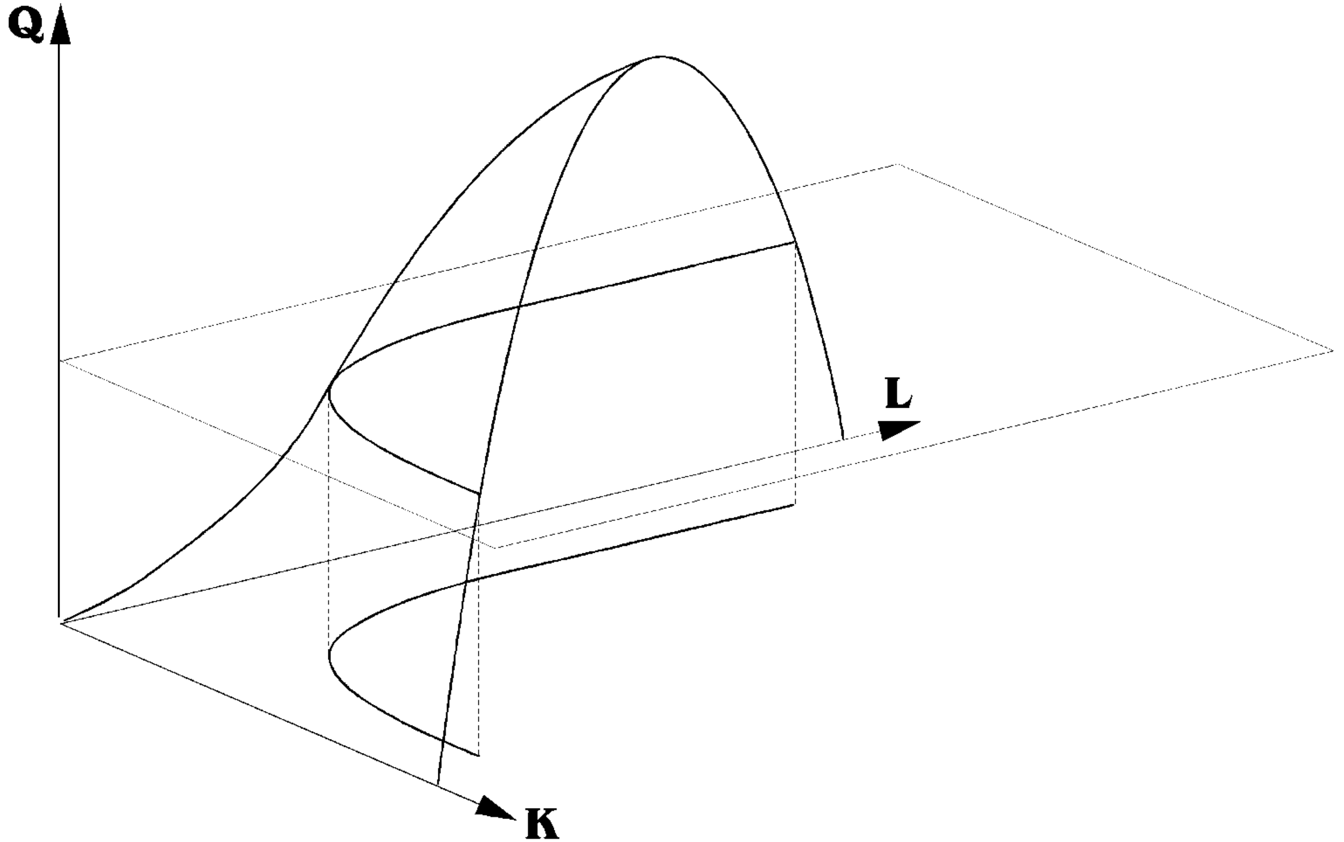
- Inputs include labour, machinery, buildings, raw materials, and energy.
- Suppose our firm uses inputs to make spoons. This is an engineering and management problem.
- Making spoons is largely a matter of technology and on-the-job experience.

The production function

- The production function summarizes technically efficient ways to combine inputs to produce output.
- A production technique is **technically efficient** if there is no other way to make a given output using less of one input and no more of the other inputs.
- The **production function** is the set of all technically efficient techniques.

Production functions

- Production function (in theory): Q = f(L,K,A,E)
- Simplified version: Q = f(K,L)



Wasteful production methods

- Since profit maximizing firms are not interested in wasteful production methods, we restrict our attention to those that are technically efficient.
- To make a spoon, method ***A*** needs 2 workers and 1 machine, but method ***B*** needs 2 workers and 2 machines.
- Method ***B*** is less efficient than method ***A***. It uses more machines, but the same labour, to make the same output. Method ***B*** is not one of the efficient methods listed in the production function.

Example: A production function

Output	Capital input	Labour input
100	4	4
100	2	6
106	2	7
200	4	12

- The table to the left shows some technically efficient methods in the production function. The first two rows show two ways to make 100 spoons: 4 machines and 4 workers, or 2 machines and 6 workers.
- Beginning from the latter, the third row shows the effect of adding an extra worker. Output rises by 6 spoons.
- The last row shows that doubling both inputs in the second row also doubles the output, though this need not be so: overcrowding a small factory can slow people down.

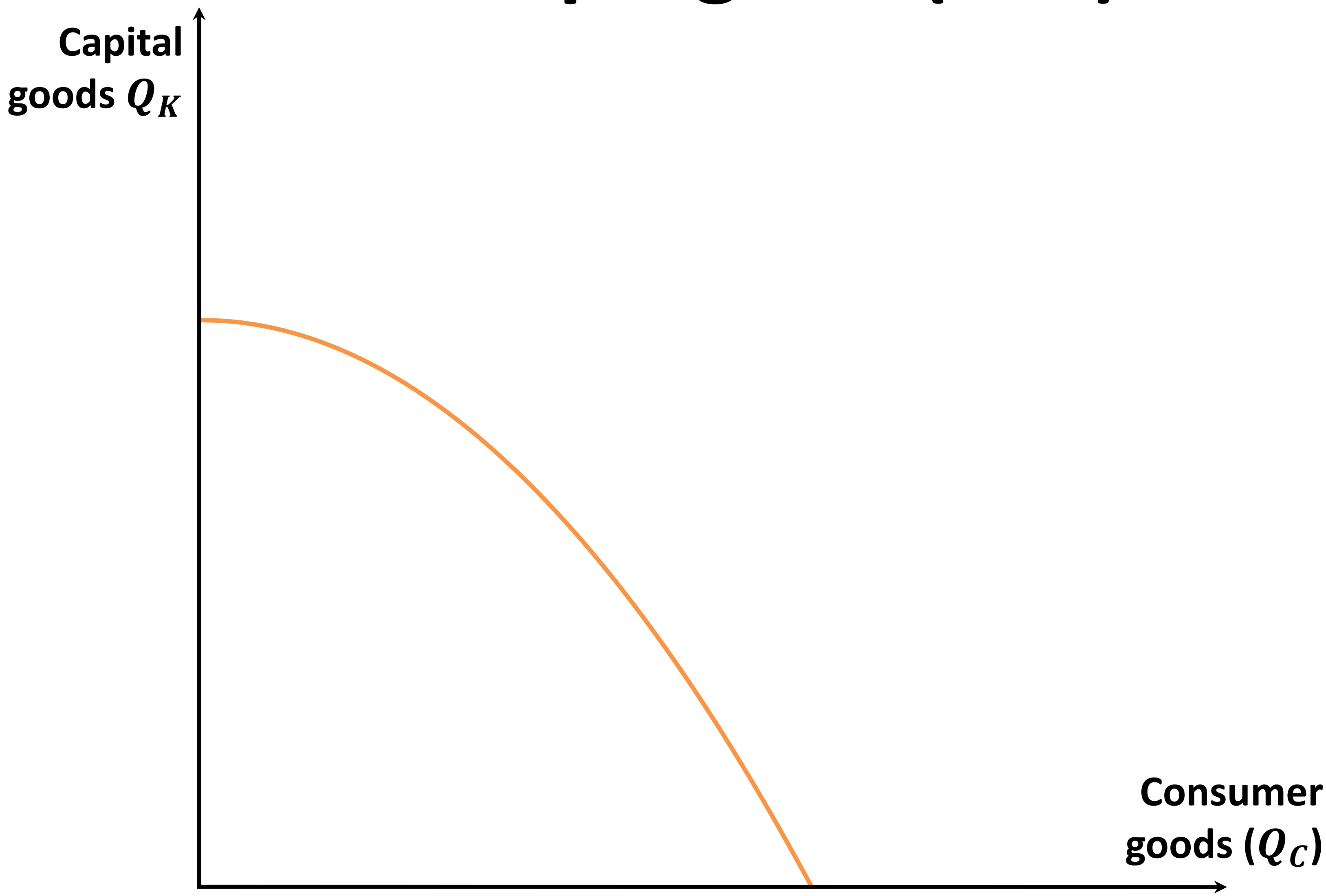
Why do firms need engineers?

- The previous table could be enlarged to include other combinations of labour and capital also technically efficient.
- A firm discovers its production function, the complete set of technically efficient production techniques, by asking its engineers, designers, and time-and-motion experts; and by trial and error.

Technique, technology, and technical progress

- A **technique** is a particular way to combine inputs to make output.
- **Technology** is the list of all known techniques.
- **Technical progress** is a new technique allowing a given output to be made with fewer inputs than before.

Technical progress (PPF)



Technical progress

- A method previously technically efficient may become inefficient after a technical advance allows a better production technique.
- Technical progress alters the production function.
- For now, we assume a given technology and a given production function.

- The production function relates volumes of inputs to volume of output. To get costs, we also need to know input prices.
- Consider the lowest-cost way to make 100 spoons. Assume that there are two technically efficient techniques, the first two rows of our previous table, reproduced as the first two columns of the table below and labelled techniques ***A*** and ***B***.

Technique	Capital input	Labour input	Rental per machine	Wage per worker	Capital cost	Labour cost	Total cost
A	4	4	€320	€300	€1280	€1200	€2480
B	2	6	€320	€300	€640	€1800	€2440

Economically efficient production methods

- It costs €320 to rent a machine, and €300 to hire a worker.
- To make 100 spoons, the table on the previous slide shows that the total cost is €2480 with technique ***A*** and €2440 with technique ***B***.
- The firm chooses ***B***. 100 spoons at a total cost of €2440 is one point on the total cost curve for spoons. It is the **economically efficient** (lowest-cost) production method at the going rental and wage rates.

Deriving the total cost curve

- To get the whole **total cost** curve, we repeat the calculation for each output. The production function tells us the inputs needed by each technique. Using input prices, we calculate the cost using each technique, and choose the lowest-cost production method.
- Joining up these points we get the total cost curve, which may switch from one production technique to another at different outputs. From the total cost curve we calculate the marginal cost curve – the rise in total cost at each output when output is increased by one more unit.

Factor intensity

- A technique using a lot of capital and little labour is ‘capital-intensive’, one using a lot of labour but relatively little capital is called ‘labour-intensive’.
- In our previous table, technique ***A*** is more capital-intensive and less labour-intensive than technique ***B***. (The ratio of capital input to labour is 1 in technique ***A*** but only one-third in technique ***B***.)

Factor prices and relative prices

- At the **factor prices** (prices per unit input) in our previous table, the more labour-intensive technique is cheaper.
- Suppose the wage rises from €300 to €340: labour is dearer but the rental on capital is unchanged.
- The **relative price** of labour has risen.

Effects of a price change

- We ask two questions: First, what happens to the total cost of making 100 spoons?
- Second, is there any change in the preferred technique?
- The table below recalculates production costs at the new factor prices.

Technique	Capital input	Labour input	Rental per machine	Wage per worker	Capital cost	Labour cost	Total cost
A	4	4	€320	€340	€1280	€1360	€2640
B	2	6	€320	€340	€640	€2040	€2680

Effects of a price change (2)

- Because both techniques use some labour, the total cost of making 100 spoons by each technique rises.
- Repeating this argument at all output, the total cost curve must shift upwards when the wage rate (or the price of any other input) rises.
- In this example, the rise in the relative price of labour leads the firm to switch techniques: it switches to the more capital-intensive technique ***A***.

Long-run analysis

- Faced with an upward shift in its demand and MR curves, a firm will expand output (see the handouts of the previous lecture).
- However, adjustment takes time. Initially, the firm can get its existing workforce to do overtime. In the long run, the firm can vary its factory size, switch techniques of production, hire new workers and negotiate new contracts with suppliers of raw materials.

Long run and short run

- The **long run** is the period long enough for the firm to adjust **all its inputs** to a change in conditions.
- In the **short run** the firm can make only **partial** adjustment of its inputs to a change in conditions.

- The firm may be able to alter the shift length at once.
- Hiring or firing workers takes longer, and it might be years before a new factory is designed, built, and operational.
- For now, we will only analyse long-run cost curves, when the firm can make all the adjustments it desires.

Long-run total and marginal costs

- **Long-run total cost** is the minimum cost of producing each output level when the firm can adjust all inputs.
- **Long-run marginal cost** is the rise in long run total cost if output rises permanently by one unit.

Alternatively, it may be defined as the derivate of the total cost function LTC(q):

$$\mathrm{LMC} = \frac{\partial \mathrm{LTC(q)}}{\partial \mathrm{q}}$$

Long-run costs

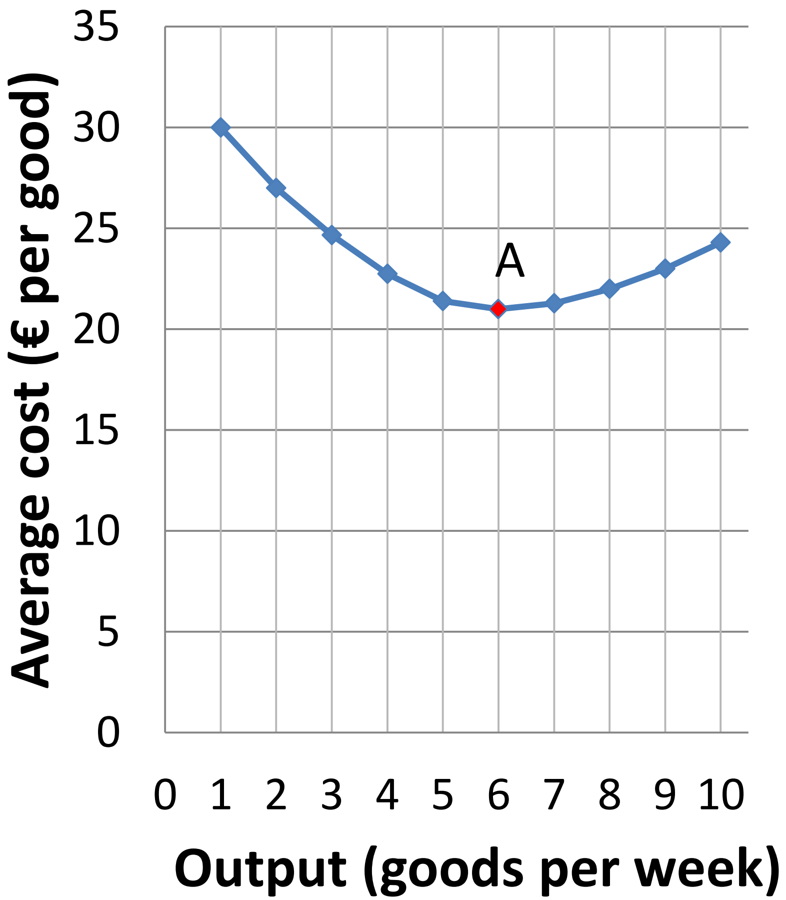
(1) Output	(2) Total cost (€)	(3) Marginal cost (€)	(4) Average cost (€)
0	0		
1	30	30	30
2	54	24	27
3	74	20	24.67
4	91	17	22.75
5	107	16	21.4
6	126	19	21
7	149	23	21.29
8	176	27	22
9	207	31	23
10	243	36	24.3

- The table to the left shows long-run total costs LTC and long-run marginal costs LMC of making each output.
- Since there is always an option to close down entirely, the LTC of producing zero output is zero. LTC describes the eventual costs after all adjustments have been made.
- LTC must rise with output: higher output always costs more to produce. LMC is always positive.

Long-run average cost

- Can large firms produce goods at a lower unit cost than small firms? Might it be a disadvantage to be (too) large?
- To answer these questions, we need to think about average cost per unit of output.
- **Long-run average cost** (LAC) is the total cost (LTC) divided by the level of output (Q).

The LAC curve (figure 1)

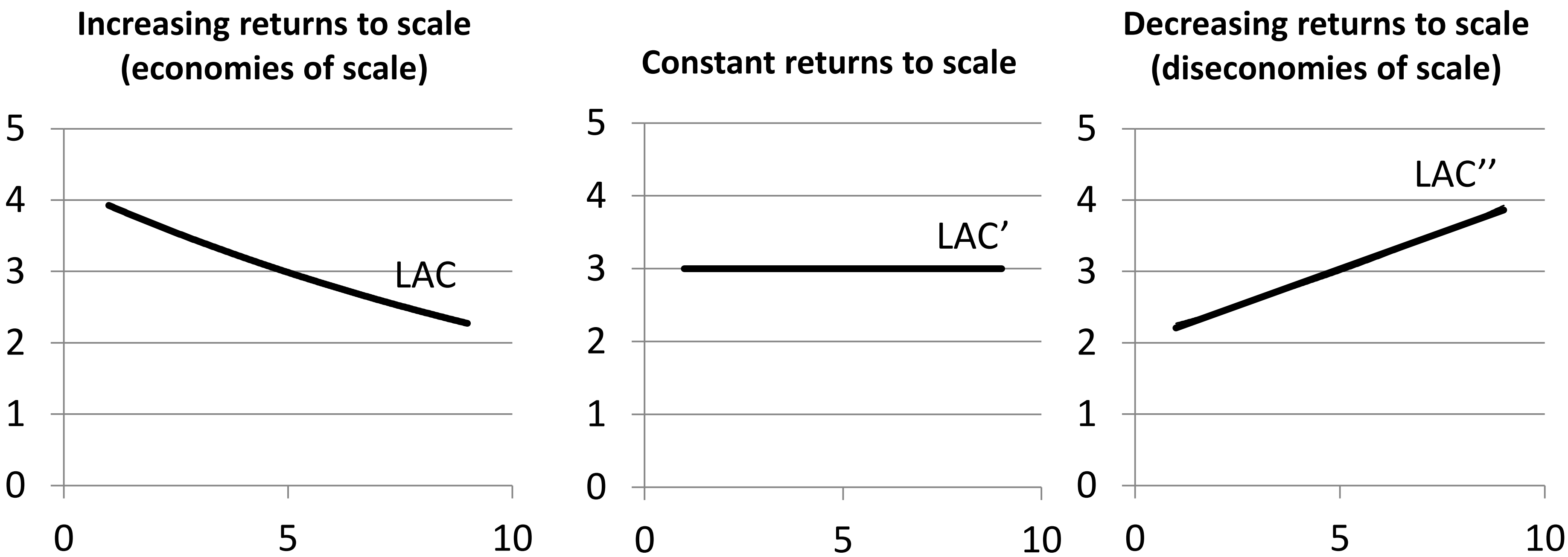


- Our previous table showed long-run average cost LAC (column 2 divided by column 1). These LAC data are plotted in the figure to the left (figure 1).
- Average cost starts out high, then falls, then rises again. This common pattern of average costs is called the U-shaped average cost curve. To see why U-shaped average cost curve is common in practice, we examine ‘returns to scale’.

Returns to scale

- **Economies of scale (or increasing returns to scale)** mean long-run average cost falls as output rises.
- **Diseconomies of scale (or decreasing returns to scale)** mean long-run average cost rises as output rises.
- **Constant returns to scale** mean long-run average costs are constant as output rises.

Returns to scale and long-run average cost (LAC) curves



- The three LAC curves show the relationship between returns to scale and the shape of the LAC curve.
- When LAC is declining, average costs of production fall as output increases and there are economies of scale.
- When LAC is increasing, average costs of production increase with higher output, and there are decreasing returns to scale.
- The intermediate case, where average costs are constant, has constant returns to scale.

Returns to scale (2)

- In figure 1 the U-shaped average cost curve had scale economies up to point A, were average cost was lowest. At higher outputs there were diseconomies of scale.
- Why are there scale economies at low output levels but diseconomies of scale at high output levels? We draw a cost curve for given input prices. Changes in average costs as we move along the LAC curve cannot be explained by changes in factor prices. (Changes in factor prices ***shift*** cost curves.)
- The relationship between average costs and output as we move along the LAC curve depends on the technical relation between physical quantities of inputs and output, summarized in the production function.

Returns to scale (3)

Formally, „returns to scale”, refers to changes in output (quantity, q) resulting from a proportional change (λ) in all inputs:

- If output increases by less than that proportional change, there are decreasing returns to scale:
  e.g. $Q = \sqrt[3]{KL} \quad Q(\lambda K, \lambda L) = \sqrt[3]{(\lambda K)(\lambda L)} = \lambda^{2/3} Q(K, L)$

- If output increases by that same proportional change, there are constant returns to scale:
  e.g. $Q = \sqrt{KL} \quad Q(\lambda K, \lambda L) = \sqrt{(\lambda K)(\lambda L)} = \lambda Q(K, L)$

- If output increases by more than that proportional change, there are increasing returns to scale (economies of scale):
  e.g. $Q = KL \quad Q(\lambda K, \lambda L) = (\lambda K)(\lambda L) = \lambda^2 Q(K, L)$

Increasing returns to scale mean that one (big) company can produce more efficiently than more than one (smaller) firms utilizing the same factors of production → emergence of natural monopolies

Indivisibilities

- There are three reasons for economies of scale. The first is indivisibilities in the production process, a minimum quantity of inputs required by the firm to be in business at all whether or not output is produced.
- These are sometimes called quasi-fixed costs, because they do not vary with output level. To be in business a firm requires a manager, a telephone, an accountant, a market research survey. The firm cannot have half a manager and half a telephone merely because it wishes to operate at low output levels.

Quasi-fixed costs

- Beginning from small output levels, these costs do not initially increase with output.
- The manager can organize three workers as easily as two. As yet there is no need for a second telephone.
- There are economies of scale because these costs can be spread over more units of output as output is increased, reducing average cost per unit of output.
- However, as the firm expands further, it has to hire more managers and telephones and these economies of scale die away.
- The average cost curve stops falling.

Specialization

- The second reason for economies of scale is specialization.
- A sole trader must undertake all the different tasks of the business.
- As the firm expands and takes on more workers, each worker can concentrate on a single task and handle it more efficiently.

Machinery

- The third reason for economies of scale is closely related. Large scale is often needed to take advantage of better machinery.
- No matter how productive a robot assembly line is, it is pointless to install one to make five cars a week. Average costs would be enormous.
- However, at high output levels the machinery cost can be spread over a large number of units of output and this production technique may produce so many cars that average costs are low.

Managerial diseconomies of scale

- Beyond some output, the U-shaped average cost curve turns up again as diseconomies of scale begin.
- Management is harder as the firm gets larger: there are managerial diseconomies of scale.
- Large companies need many layers of management, themselves needing to be managed. The company becomes bureaucratic, co-ordination problems arise, and average costs begin to rise.

Other examples of diseconomies of scale

- Geography may also explain diseconomies of scale. If the first factory is located in the best site to minimize the cost of transporting goods to the market, the site of a second factory must be less advantageous.
- To take a different example, in extracting coal from a mine, a firm will extract the easiest coal first. To increase output, deeper coal seams have to be worked and these will be more expensive.

The shape of the average cost curve

- As output increases, the shape of the average cost curve thus depends on two things: how long economies of scale persist, and how quickly the diseconomies of scale set in.
- The balance of these two forces varies from industry to industry and firm to firm.

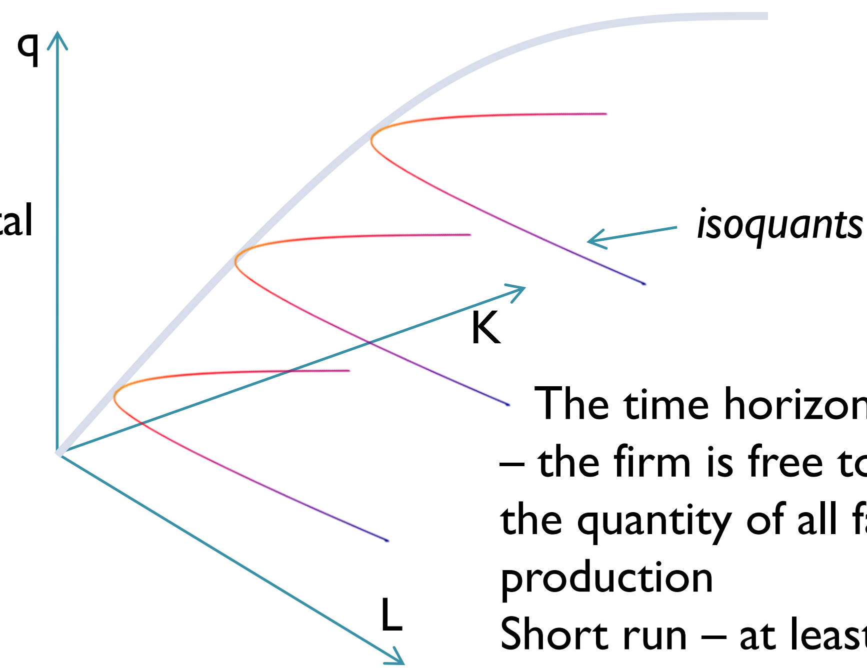
Costs and supply (Appendix)

Microeconomics

The production function

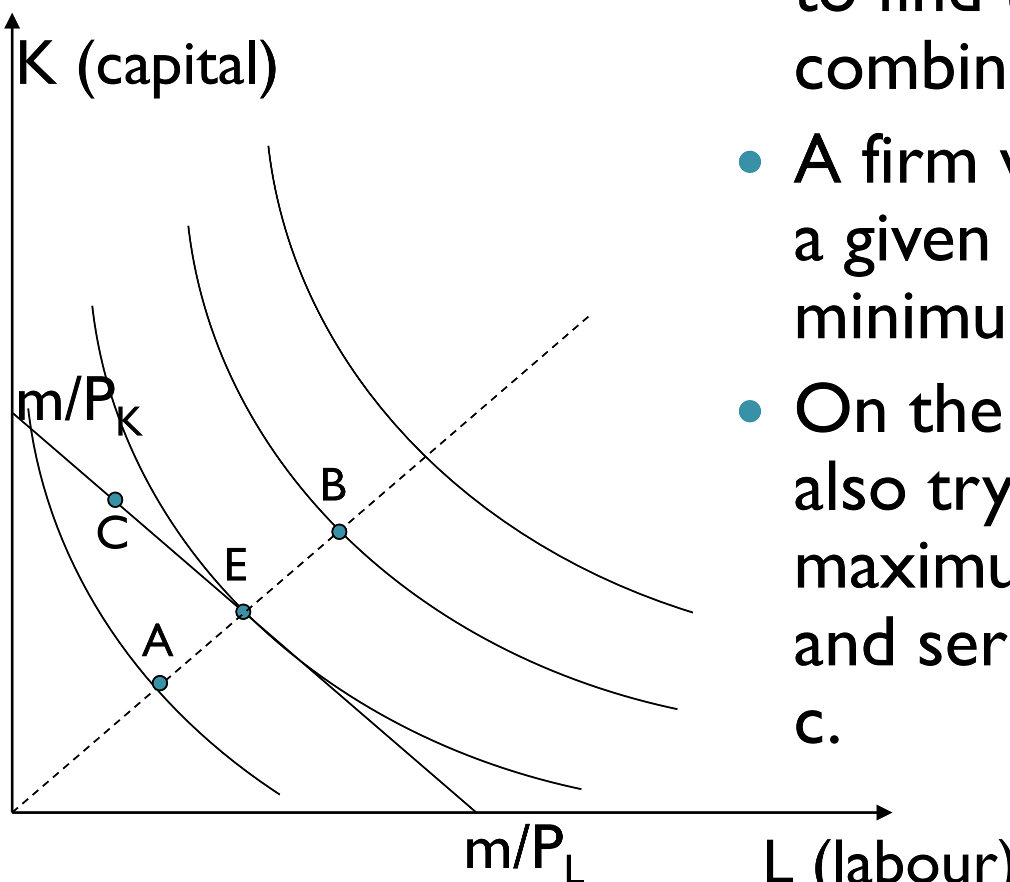
- The production function ($Q = f(x_1, x_2, \ldots)$) shows how many goods or services a firm can produce utilizing its resources (factors of production) – the maximum output possible from a given set of inputs.

In our examples
K – Capital [← das Kapital (German)]
L – Labour
q – Quantity of products



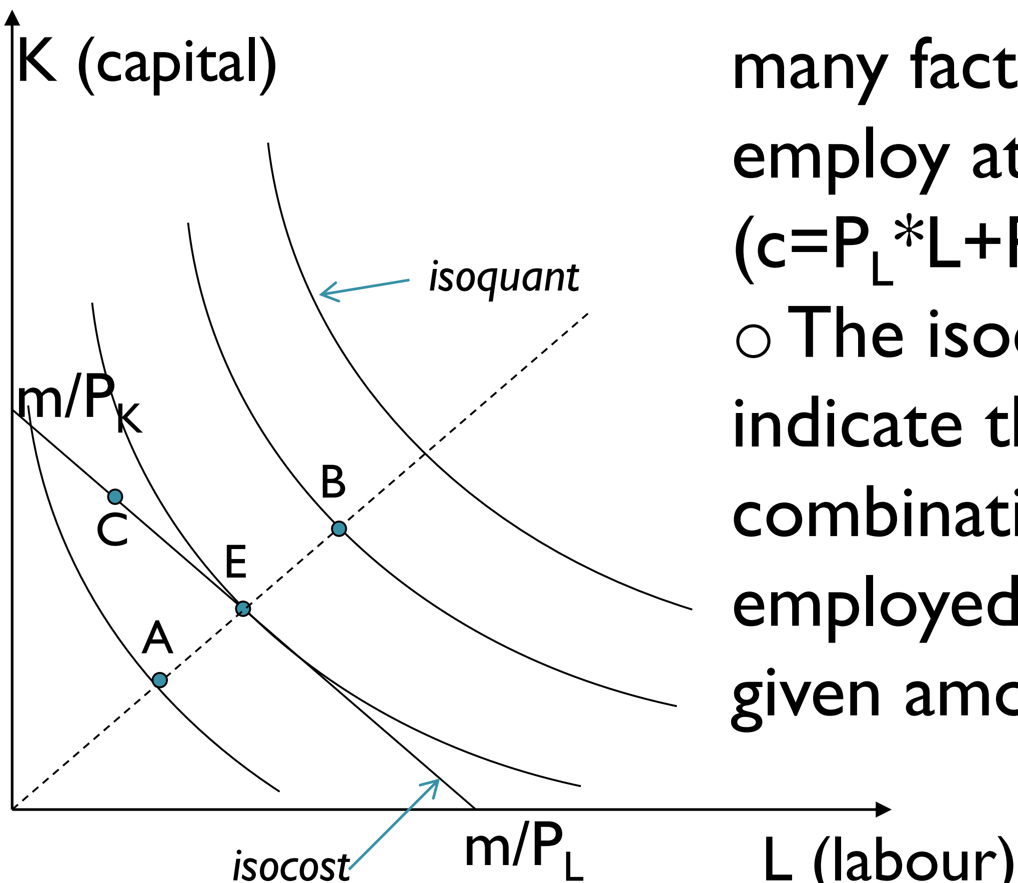
The time horizon: Long run – the firm is free to choose the quantity of all factors of production
Short run – at least one of these factors is fixed

The optimal combination of factors of production



- In the long run, the firm tries to find the optimal combination of factors:
- A firm would try to produce a given quantity „q" at the minimum cost (LTC(q)).
- On the other hand it would also try to produce the maximum amount of goods and services at a given cost c.

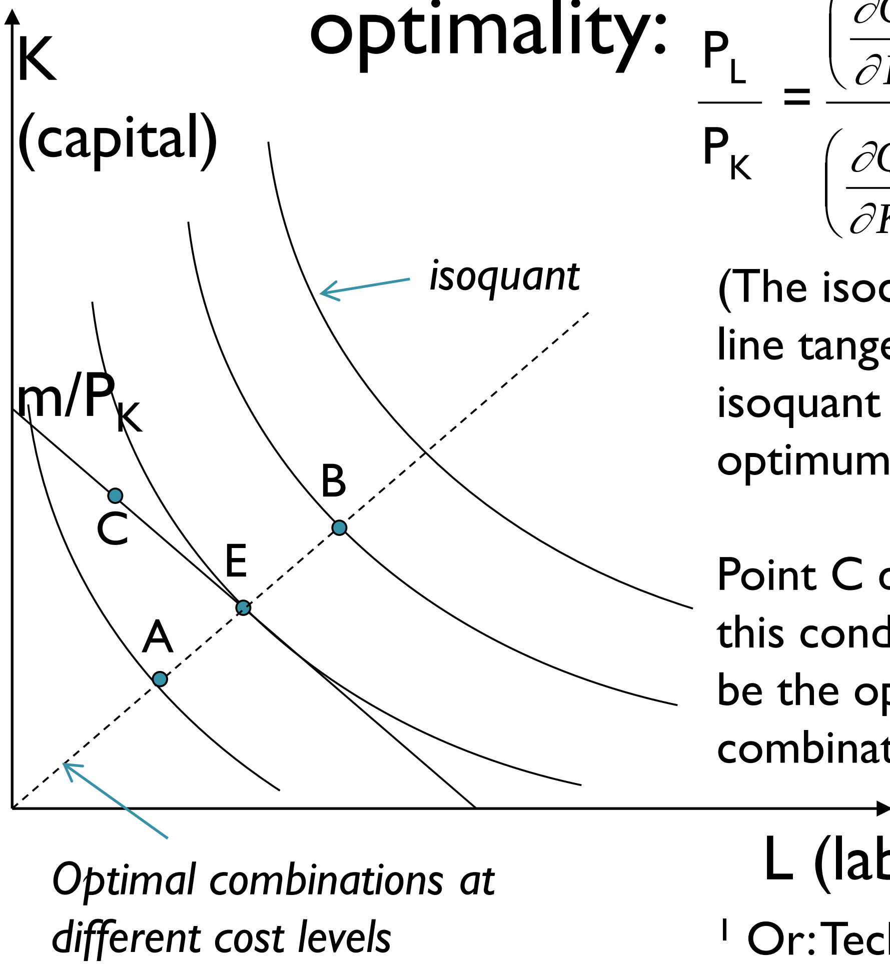
The optimal combination of factors of production II.



- The isocost lines show how many factors the firm can employ at a given cost level c. ($c=P_L*L+P_K*K$)
- The isoquant curves indicate the factor combinations that can be employed to produce the given amount of goods.

The optimal combination of factors of production III.

The necessary condition for optimality: $\frac{P_L}{P_K} = \frac{\left(\frac{\partial Q}{\partial L}\right)}{\left(\frac{\partial Q}{\partial K}\right)}$ = MRTS*



(The isocost line is the line tangent to the isoquant at the optimum point)

Point C does not satisfy this condition so it can't be the optimal combination.

*MRTS: the Marginal Rate of Technical Substitution

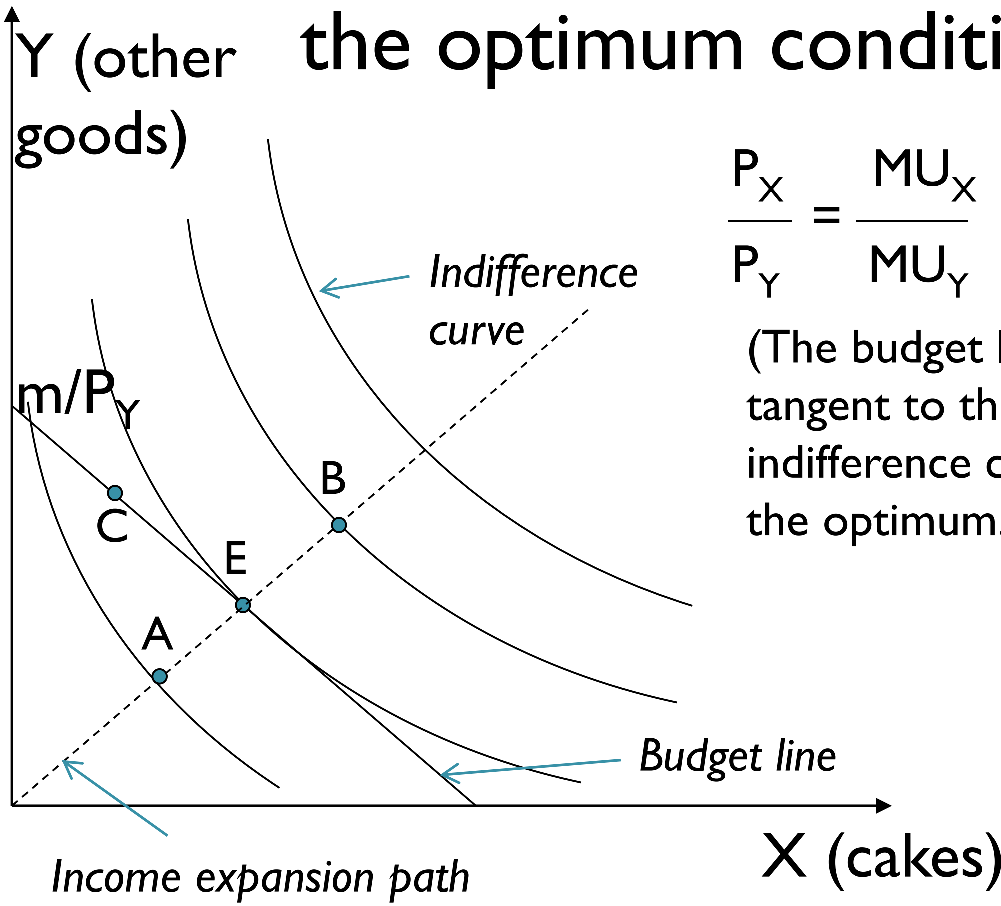
The optimum point is at „E"

[1] Or: Technical Rate of Substitution (TRS)

Consumer behaviour

Reminder

The optimum condition is similar to the optimum condition of the firm:



$$\frac{P_X}{P_Y} = \frac{MU_X}{MU_Y} = MRS^*$$

(The budget line is a tangent to the indifference curve at the optimum.)

MRS: Marginal Rate of Substitution

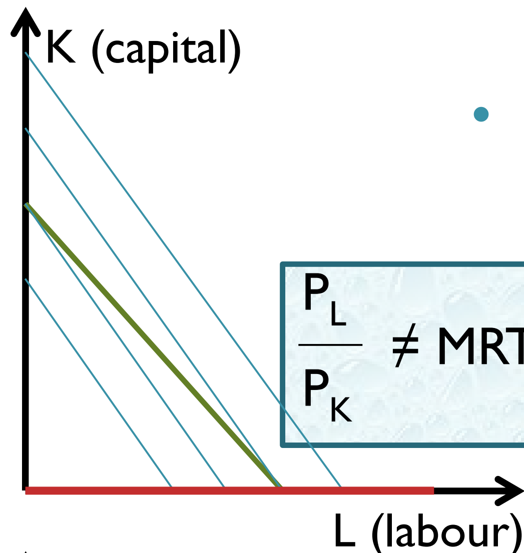
The optimum point is at ”E”

„Special technologies"

isocost

isoquant

growth path

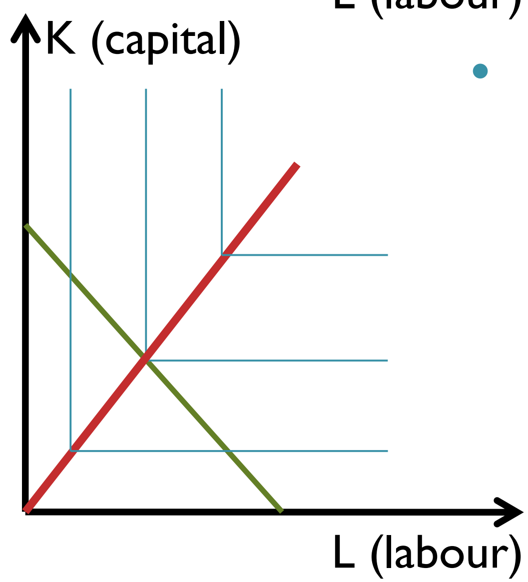


- The factors of production are perfect substitutes → the firm would only use one input type (here: labour)

  $Q=\alpha L+\beta K$

  if $\alpha/\beta > pL/pK$, the firm would only use labour;

  if $\alpha/\beta < pL/pK$, the firm would only use capital.



- Leontief production function: The factors of production are perfect complements → the factors must be used in fixed proportions

  $Q=\min\{\alpha L; \beta K\}$

  if the opt. condition $L=\beta/\alpha K$

  is not satisfied, the company is wasting its resources.

Leontief production function

- This type of production function is characteristic of the chemical industry and chemical reactions

- Example: The direct synthesis of hydrogen chloride using hydrogen and pure chlorine gas

$$H_2 + Cl_2 \rightarrow 2HCl$$

- Prod. func.: $Q_{HCl} = 2\min\{Cl_2; H_2\}$

- The inputs of this production function are hydrogen and chlorine gas, the output is hydrogen chloride (but: several other inputs might be needed, e.g. a factory, facilities (capital), chemists/technicians (labour)).

Differences between utility functions and production functions

- According to the theory of ordinal utility, we can only determine whether one bundle of goods is preferred to another (by a consumer), but the nominal value of utility has no significance. (The utility functions $U(x,y)=xy$; $U(x,y)=(xy)^2$ and $U(x,y)=\ln(x)+\ln(y)$ are considered equivalent, describing the same preference map.)
- On the other hand the isoquants of production represent the quantity of goods produced by the firm. Production functions $Q(L,K)=(KL)^{0,5}$ and $Q(L,K)=KL$ are not considered equivalent, the technology represented by the latter is more efficient and therefore preferred by the firm.

➔ Returns to scale

Returns to scale in practice

- To gather evidence on returns to scale we can talk to design engineers to see how production costs vary with output.
- Many studies of manufacturing industries confirm that scale economies continue over a wide range of output.
- The long-run average cost curve slopes down, albeit at an ever decreasing rate.

Manufacturing and service sector industries

- Scale economies in manufacturing industries are substantial.
- However, there are many industries, even in the manufacturing sector, where minimum efficient scale (MES) for a firm is small relative to the whole market and average costs are only a little higher if output is below minimum efficient scale.
- **Minimum efficient scale** is the lowest output at which the LAC curve reaches its minimum.
- For some industries, particularly personal services (hairdressers, doctors, etc.), economies of scale run out at quite low output levels.

Costs and supply 2:
A firm’s output decisions
in the long run and in the short run

Microeconomics

Reminder

Long-run costs

(1) Output	(2) Total cost (€)	(3) Marginal cost (€)	(4) Average cost (€)
0	0		
1	30	30	30
2	54	24	27
3	74	20	24.67
4	91	17	22.75
5	107	16	21.4
6	126	19	21
7	149	23	21.29
8	176	27	22
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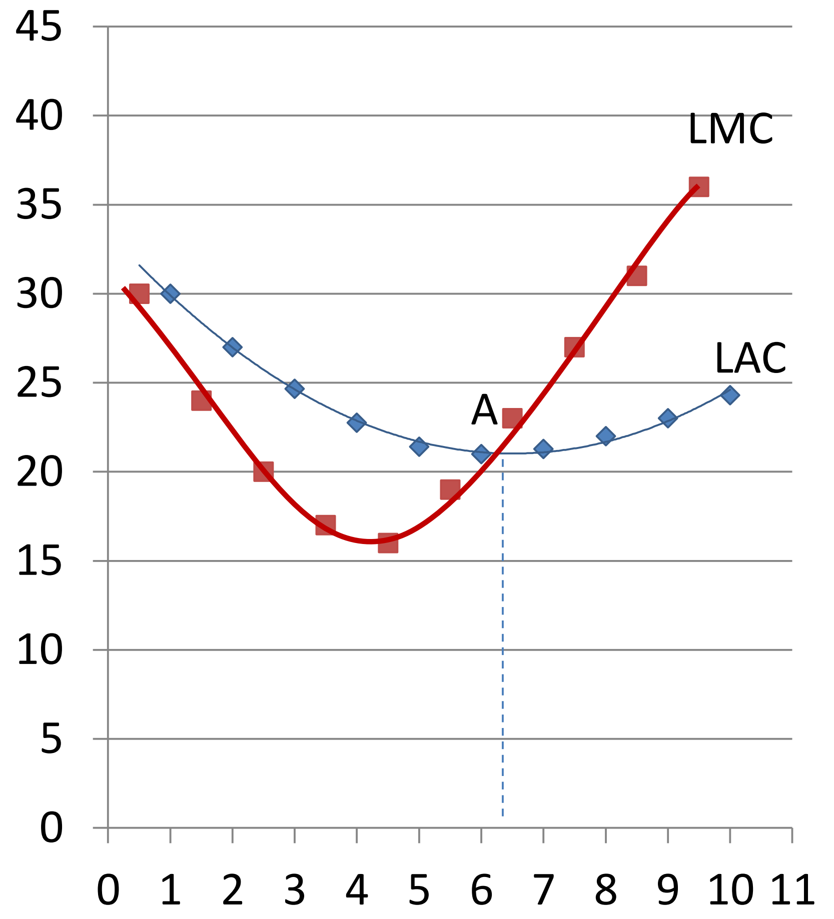
10	243	36	24.3



Average cost and marginal cost

- In the figure on the previous slide, we connect the different cost measures (LTC, LAC, LMC), whose behaviour is closely related.
- At each output, LAC is total cost divided by output. To stress that marginal cost is incurred by moving from one output level to another, LMC is plotted at points halfway between the corresponding outputs.
- The LMC of €30 for the first unit is plotted at the output halfway between 0 and 1.

Average cost and marginal cost (2)

- Two factors stand out from the table and diagram:

1. LAC is falling when LMC is less than LAC, rising when LMC is greater than LAC.
2. LAC is at a minimum at the output at which LAC and LMC cross.

Example: marginal and average

- Neither fact is an accident. The relation between average and marginal is a matter of arithmetic, as relevant for football as for production costs.
- A footballer with 3 goals in 3 games averages 1 goal a game. Two goals in the next game, implying 5 goals from 4 games, raises the average to 1.25 goals a game.
- In the fourth game the marginal goals were 2, raising total goals from 3 to 5. Because the marginal score exceeds the average score in previous games, the extra game must drag up the average.

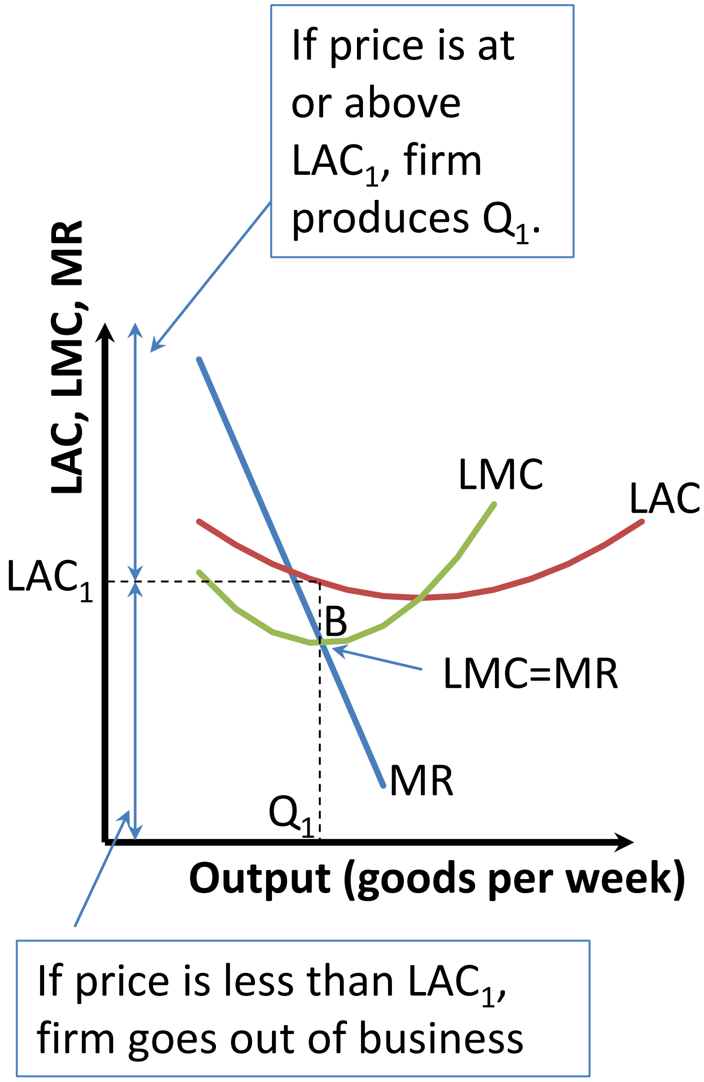
Marginal and average cost

- The same holds for production costs. When the marginal cost of the next unit exceeds the average costs of the existing units, making the next unit must raise average costs.
- If the marginal cost of the next unit lies below the average cost of existing units, an extra unit of production drags down average costs.
- When marginal and average cost are equal, adding a unit leaves average cost unchanged.

LAC_{min} is at q, where LMC=LAC

- In figure 1 average and marginal cost curves cross at point A, which must be the point of minimum average cost.
- To the left of A, LMC is below LAC so average cost is still falling. To the right of A, LMC is above LAC so average cost is rising. Average cost is lowest at A.
- The marginal cost curve crosses the average cost curve from below at the point of minimum average cost. It is true both for the relationship between LMC and LAC and for the relationship between short-run average cost (SAC) and short-run marginal cost (SMC)

The firm's long-run output decision



- The figure to the left shows smooth LAC and LMC curves for a firm not restricted to produce integer units of output. It also shows the firm's marginal revenue (MR) schedule.
- We already know that the output of maximum profit, or minimum loss is at B, the output at which marginal revenue equals marginal cost.
- The firm then checks whether it makes profits or losses at this output. It should not stay in business if it makes losses for ever.

Total profit and average profit

- Total profit is average profit per unit of output, multiplied by output.
- Total profit is positive only if average profit is positive.
- Average profit is average revenue minus average cost.
- But average revenue is simply the price for which each unit of output is sold.

Break-even point

- Hence, if long-run average costs at B exceed the price for which output Q_1 is sold, the firm makes losses in the long run and should close down.
- If, at this output, price equals LAC, the firm just breaks even.
- If price exceeds LAC at this output, the firm makes long-run profits and happily remains in business.

The firm's long-run output decision (summary)

- First, we use the **marginal condition** (LMC=MR) to find the best output *provided* the firm stays in business.
- Then, we use the **average condition** (comparing long-run average cost at this output with the price or average revenue received) to see if the best positive output yields a profit or a loss.

Short run: Fixed and varied factors of production

- The **short run** is the period in which the firm cannot fully adjust to a change in conditions.
- In the short run the firm has some ($k \geq 1$) fixed factors of production.
- A **fixed factor of production** is an input that cannot be varied.
- A **variable factor** can be varied, even in the short run.

Short run in different industries

- How long this short run lasts depends on the industry.
- It might take ten years to build a new power station but only a few months to open new restaurant premises if an existing building can be bought, converted and decorated.

Fixed costs

- The existence of fixed factors in the short run has two implications. First, in the short run the firm has some fixed costs.
- **Fixed costs** do not vary with output.
- These fixed costs must be borne even if output is zero. If the firm cannot quickly add to or dispose of its existing factory, it must still pay depreciation on the building and meet the interest cost of the money it originally borrowed to buy the factory.
- Second, because in the short run the firm cannot make all the adjustments it would like, its short-run costs must exceed its long-run costs.

Short-run costs of production

(1) Output	(2) SFC	(3) SVC	(4) STC	(5) SMC
0	30	0	30	N/A
1	30	22	52	22
2	30	38	68	16
3	30	48	78	10
4	30	61	91	13
5	30	79	109	18
6	30	102	132	23
7	30	131	161	29
8	30	166	196	35
9	30	207	237	41
10	30	255	285	48

- The table to the left presents data on short-run costs.
- The second column shows the fixed costs, which are independent of the output level.
- The third column shows the variable costs.
- The fourth column shows total costs (fixed + variable).
- The final column shows the marginal costs.

Variable costs

- **Variable costs** change as output changes.
- Variable costs are the costs of hiring variable inputs, typically labour and raw materials.
- Firms may have long-term contacts with workers and material suppliers, which reduce the speed at which these inputs can be adjusted. Yet most firms retain some flexibility through overtime and short time, hiring or non-hiring of casual and part-time workers, and raw material purchases in the open market to supplement contracted supplies.

Total and marginal costs

- The fourth column of the table shows short-run total costs:

  Short-run total costs = Short-run fixed costs (SFC) +
  (STC) Short-run variable costs (SVC)

- The final column shows short-run marginal costs SMC. Since fixed costs do not rise with output, SMC is the rise both in short-run total costs and in short-run variable costs as output is increased by one unit.

Costs and output

- Whatever the output, fixed costs are €30 per week. Marginal costs are always positive.
- Short-run total costs rise steadily as output rises. Extra output adds to total cost, and adds more the higher is the marginal cost.
- In the last column of our first table, as output increases marginal costs first fall then rise again.

The shape of the SMC curve

- The short-run marginal cost curve has the same shape as the long-run marginal cost curve, but for a different reason.
- In the long run the firm can vary all factors freely.
- As output expands, the firm enjoys some scale economies, then diseconomies of scale set in.
- The short-run marginal cost curve assumes that there is at least one fixed factor, probably capital.
- Suppose there are two inputs in the short run, fixed capital and variable labour.
- To change output as we move along the short-run marginal cost curve, the firm adds ever-increasing amounts of labour to a given amount of plant and machinery.
- This explains the shape of the short-run marginal curve.

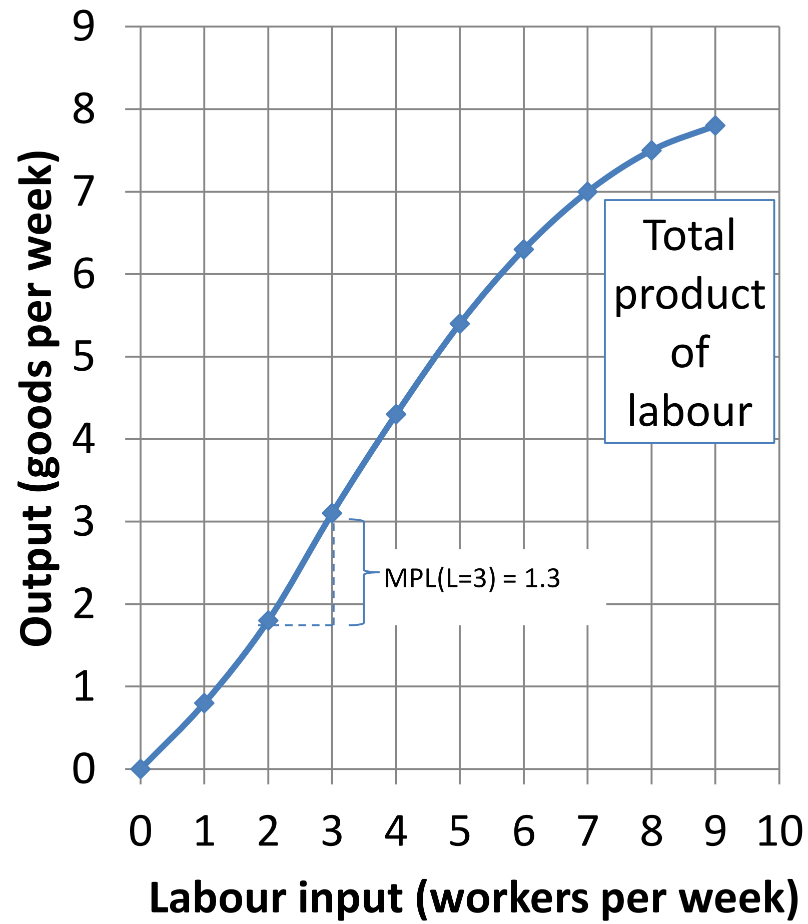
Total and marginal products of labour

Labour input (workers)	Output (total product)	Marginal product of labour
1	0.8	0.8
2	1.8	1.0
3	3.1	1.3
4	4.3	1.2
5	5.4	1.1
6	6.3	0.9
7	7.0	0.7
8	7.5	0.5
9	7.8	0.3

- The table to the left shows how output rises as variable labour input is added to a fixed quantity of capital.
- With no workers output is zero.
- The first worker raises output by 0.8 units.

The marginal product of labour

- The **marginal product** of a variable factor is the extra output from an extra unit of that input, holding constant all other inputs.
- The first worker has a marginal product of 0.8 units. The third worker has a marginal product of 1.3 units, since 2 workers produce 1.8 units but 3 workers produce 3.1 units.



Labour input and output

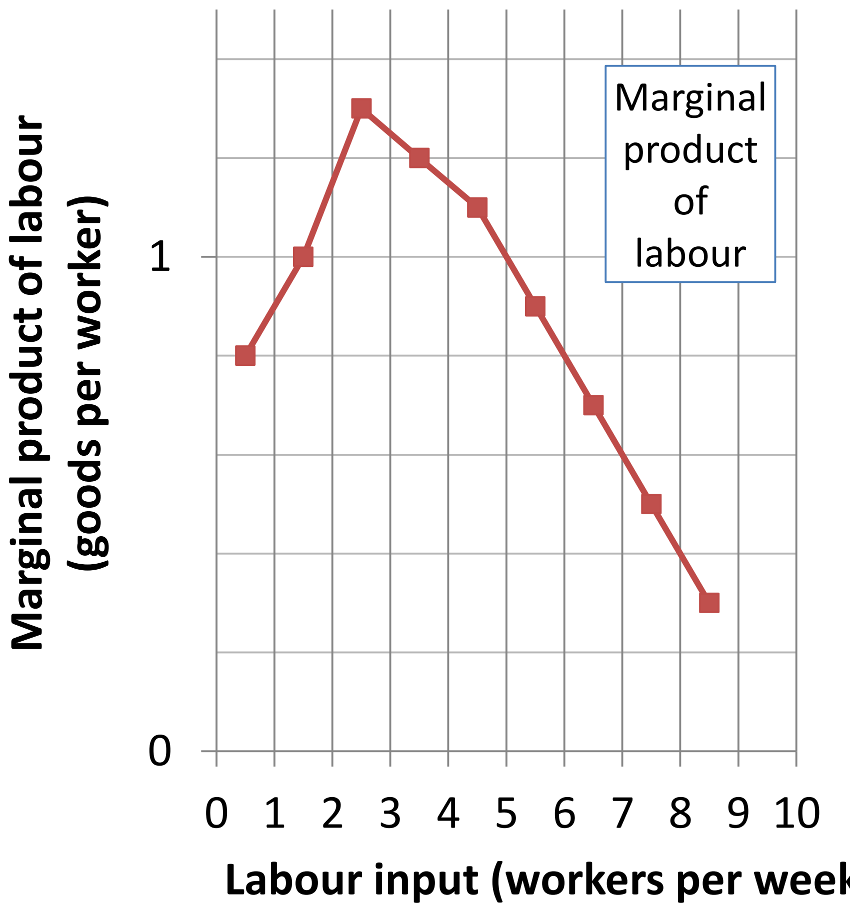
- At low levels of output and labour input, the first worker has a whole factory to work with, too many tasks to produce very much. A second worker helps, and a third helps even more.
- Suppose the factory has three machines and the three workers are each specializing in fully running one of the factory's machines. The marginal product of the fourth worker is lower.
- With only three machines, the fourth worker gets to use one only when another worker is having a rest. There is even less useful machine work for the fifth worker to do.

Law of diminishing returns

- Beyond 3 workers, the marginal product of each additional worker decreases steadily as the number of workers is increased. We say that there are diminishing returns to labour.
- Holding all factors constant except one, **the law of diminishing returns** says that, beyond some level of the variable input, further increases in the variable input lead to a steadily decreasing marginal product of that input.

Diminishing marginal returns

- This is a law about technology. Adding ever more workers to a fixed quantity of machinery gets less and less useful.
- The ninth workers main role in production is to get coffee for the others. This contributes to output but not a great deal.
- The figure to the right summarizes our discussion of marginal productivity.



The marginal product of capital

- If capital was the variable factor and labour the fixed factor, adding more and more machines to a given labour force might initially lead to large increases in output but would soon encounter diminishing returns as machines become under-utilized.
- Thus the figure on the previous slide, showing the marginal product of labour when labour is the variable factor, might also describe the behaviour of the marginal product of capital when capital is the variable factor.

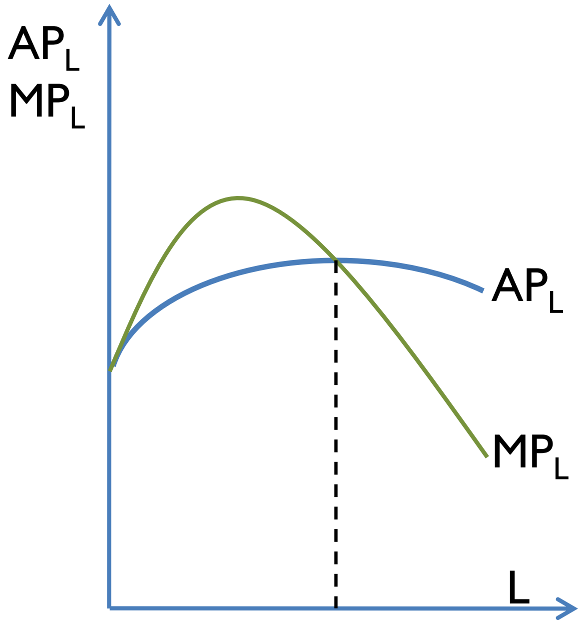
Marginal product and productivity

- Marginal product is not the everyday meaning of ‘productivity’ which refers to the average product. The average product of labour, what is most commonly meant by ‘productivity’, is total output divided by total labour input.
- If the marginal product of labour lies above the average product, adding another worker will raise the average product and ‘productivity’. When diminishing returns set in, the marginal product will quickly fall below the average product and the latter will fall if further workers are added.

The marginal product curve

- As usual, we must distinguish between movements along a curve and shifts in a curve.
- The marginal product curve is drawn for given levels of the other factors. For a higher given level of the fixed factors, the marginal product curve would be higher.
- With more machinery to work with, an extra worker will generally be able to produce more extra output than previously.

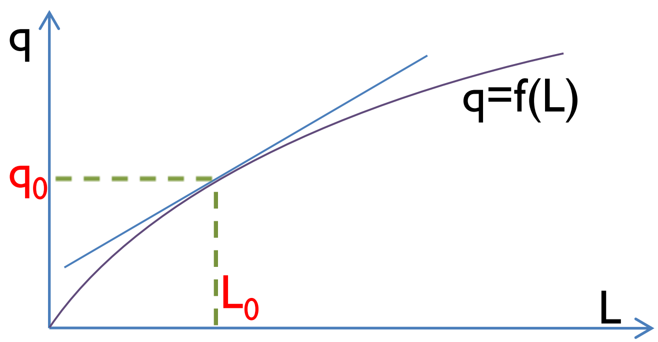
Appendix: a more precise definition of marginal productivity



$AP_L = MP_L$
Maximum of AP_L

The point of intersection between the marginal product and average product curves is the peak of the average product curve.

In the short run, the quantity of some inputs (here: capital) is fixed, the output is the function of the variable input(s) (here: labour): $Q_S = F(\overline{K}, L) = F(L)$



Marginal product: the partial derivative of the production function. The marginal product of labour (MP – *marginal product*): $MP_L = \frac{\partial Q}{\partial L}$

Average product (AP): the total output produced per unit. The average product of labour: $AP_L = \frac{Q}{L}$

Geometrically: The slope of a tangent line to the short-run production function. Approximately the change in the quantity of total product resulting from a unit change in a variable input, keeping all other inputs unchanged.

Short-run costs of production

Q	STC (SFC + SVC)	SFC	SAFC= SFC/Q	SVC	SAVC= SVC/Q	Short-run average total cost (AFC+AVC)	Short-run marginal cost
0	30	30	–	0	–	–	–
1	52	30	30	22	22	52	22
2	68	30	15	38	19	34	16
3	78	30	10	48	16	26	10
4	91	30	7.5	61	15.25	22.75	13
5	109	30	6	79	15.8	21.8	18
6	132	30	5	102	17	22	23
7	161	30	4.29	131	18.71	23	29
8	196	30	3.75	166	20.75	24.5	35
9	237	30	3.33	207	23	26.33	41
10	285	30	3	255	25.5	28.5	48

Short-run marginal costs

- The table on the previous slide shows that, as output is increased, short-run marginal costs first fall then rise.
- Every worker costs the firm the same wage. While the marginal product of labour is increasing, each worker adds more to output than the previous workers.
- Hence the extra cost of making extra output is falling. SMC is falling so long as the marginal product of labour is rising.

SMC and MP_L

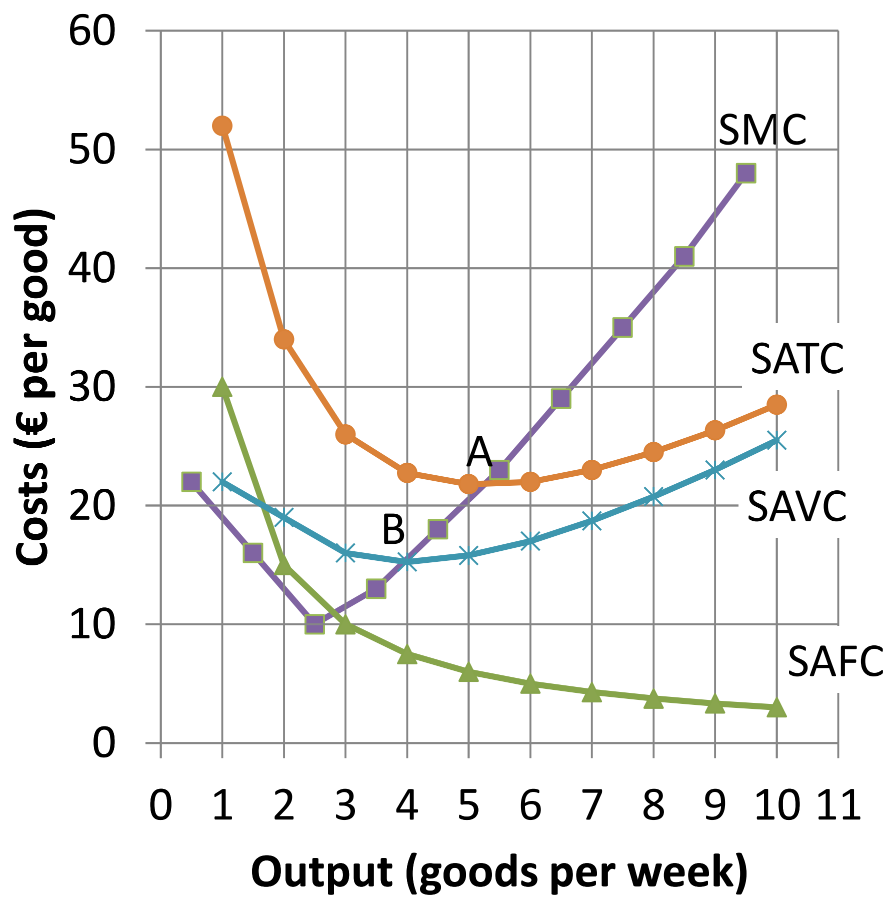
- **Short-run marginal cost** is the extra cost of making an extra unit of output in the short-run while some inputs remain fixed.
- Once diminishing returns to labour set in, the marginal product of labour falls and SMC starts to rise again. It takes successively more workers to make each extra unit of output.
- Thus the shape of the short-run marginal cost curve and hence the short-run total cost curve is determined by the shape of the marginal product curve, which in turn depends on the technology facing the firm. (With one var. input: $SMC=P_L/MP_L$)

Short-run average costs

- **Short-run average fixed cost** (SAFC) equals short-run fixed cost (SFC) divided by output.
- **Short-run average variable cost** (SAVC) equals short-run variable cost (SVC) divided by output and **short-run average total cost** (SATC) equals STC divided by output.
- Each number in the fourth and the sixth column of the table is obtained by dividing the corresponding number in the previous column by the output level.

Short run average cost and marginal cost curves

- These diagrams plot the data of our previous table. SATC is equal to SAFC plus SAVC. The shape of the SATC curve is a result of the shapes of its two components. When both SAVC and SAFC are declining so is SATC. When SAVC starts rising, the shape of SATC depends on whether SAVC is rising more rapidly than SAFC is falling.
- The relationship between marginal and average cost curves established for the long run applies also to the short-run curves. The SMC curve goes through the minimum points of both the SAVC curve (at B) and the SATC curve (at A).



Marginal and average total cost

- The shape of the SMC curve follows from the behaviour of marginal labour productivity.
- The usual arithmetical relation between marginal and average explains why SMC passes through the lowest point A on the short-run average cost curve.
- To the left of this point, SMC lies below SATC and is dragging it down as output expands. To the right of A the converse holds. That explains the shape of the SATC curve.

Marginal and average variable cost

- Variable cost is total cost minus fixed cost. Fixed cost does not change with output. Hence marginal cost also shows how much total *variable* cost is changing.
- The usual arithmetic between marginal cost and average *variable* cost must hold. Hence, SMC goes through the lowest point B on SAVC. To the left of B, SMC is below SAVC and SAVC is falling.
- To the right of B, SAVC is rising. Finally, since average total cost exceeds average variable cost by average fixed cost, SAVC lies below SATC. Point B must lie to the left of point A. That explains the shape of SAVC and its relation to SATC.

Short-run average fixed costs

- In our figure, SAFC falls steadily, because total fixed costs ('overheads') is spread over ever larger output levels, thus reducing average fixed costs.
- At each output level, SATC = SAVC + SAFC. (This follows from dividing each term in equation 'STC = SVC + SFC' [see the handouts of the previous lecture] by the output level.)

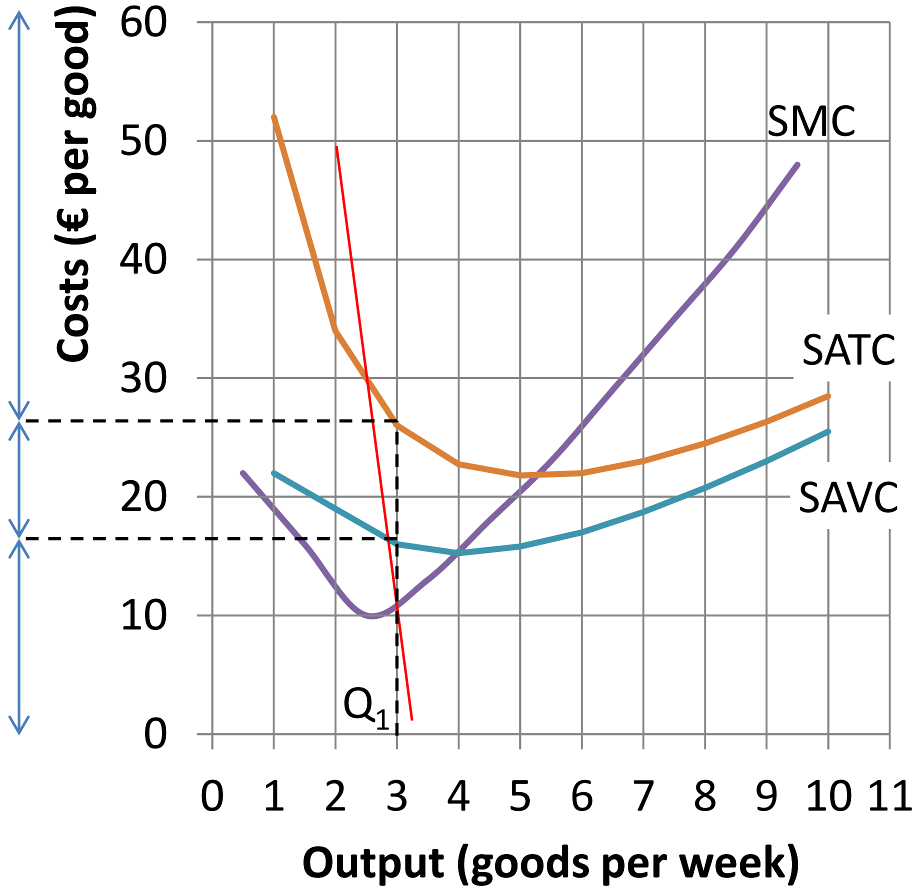
The firm's short-run output decision

The firm sets output at Q_1, where short-run marginal costs equal marginal revenue. Then it checks whether it should produce at all.

If price is above $SATC_1$, firm produces Q_1 at a profit.

If price is between $SATC_1$ and $SAVC_1$, firm produces Q_1 at a loss.

If price is less than $SAVC_1$, firm produces zero output.



At those prices, the firm is not even covering its variable costs.

Short-run profit

- Short-run marginal cost is set equal to marginal revenue to determine the output Q_1 that maximizes profits or minimizes losses.
- Next, the firm decides whether or not to produce in the short run. Profit is positive at the output Q_1 if the price *p* at which this output is sold covers average total cost.
- It is the short-run measure $SATC_1$ at output Q_1 that is relevant. If *p* exceeds $SATC_1$, the firm makes profit in the short run and produces Q_1.

Short-run losses

- Suppose *p* is less than $SATC_1$. The firm is losing money because *p* does not cover costs.
- In the long run the firm closes down if it keeps losing money.
- In the short run, even at zero output the firm must pay its fixed costs.
- The firm needs to know whether losses are bigger if it produces at Q_1 or produces zero.

Overhead and variable costs

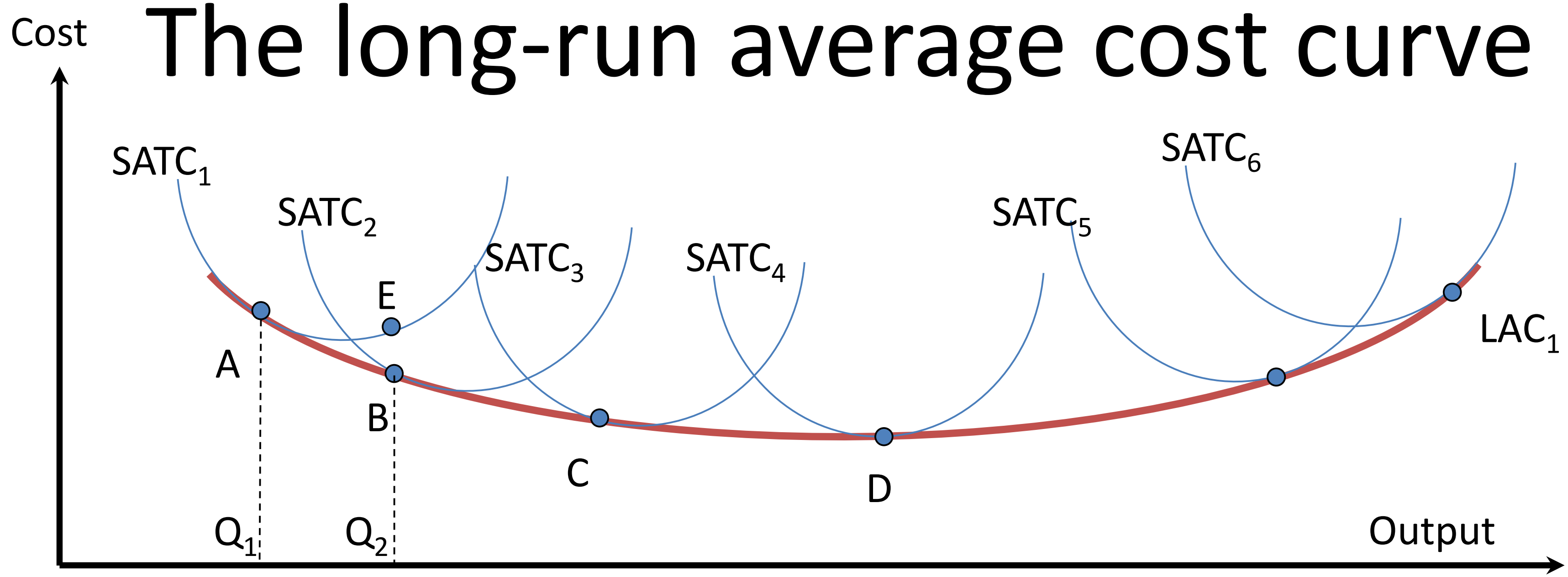
- If revenue exceeds variable cost the firm is earning something towards its overheads.
- It produces Q_1 if revenue exceeds variable cost even though Q_1 may involve losses.
- The firm produces Q_1 if p exceeds $SAVC_1$.
- If not, it produces zero.

The firm's output decisions (summary)

	Marginal condition	Check whether to produce
Short run	Choose the output at which MR = SMC	Produce this output if *p* > SAVC. Otherwise, produce zero.
Long run	Choose the output at which MR = LMC	Produce this output if *p* > LAC. Otherwise, produce zero.

Even if making losses in the short run, a firm stays in business if it covers its variable costs. In the long run it must cover all its costs to remain in business. A firm may reduce its costs in the long run, converting a short-run loss into a long-term profit.

The long-run average cost curve



Suppose the plant size is fixed in the short run. For each plant size we obtain a particular SATC curve. But in the long run, even plant size is variable. To construct the LAC curve we select at each output the plant size which gives the lowest SATC at this output. Thus points such as A, B, C and D lie on the LAC curve. Notice the LAC curve does not pass through the lowest point on each SATC curve. Thus the LAC curve shows the minimum average cost way to produce a given output when all factors can be varied, not the minimum average cost at which a given plant can produce.

The LAC curve

- The figure on the previous slide shows a U-shaped LAC curve.
- At each point on the curve the firm is producing a given output at minimum cost.
- The LAC curve describes a time scale sufficiently long that the firm can vary all factors of production, even those that are fixed in the short run.

‘Plant’ as a fixed factor

- Suppose, for convenience, that ‘*plant*’ is the fixed factor in the short run. Each point on the LAC curve involves a particular quantity of *plant*.
- Holding constant this quantity of *plant*, we can draw the short-run average total cost curve for this plant size.
- $SATC_1$ corresponds to the plant size at point A on the LAC curve, and $SATC_2$ and $SATC_3$ curves correspond to the plant size at points B and C on the LAC curve.
- In fact, we could draw an SATC curve corresponding to the plant size at each point on the LAC curve.

LAC(Q) is always lower than SATC(Q)

- By definition, the LAC curve shows the least-cost way to make each output when all factors can be varied.
- B is the least-cost way to make an output Q_2. It *must* be more costly to make Q_2 using the wrong quantity of *plant*, e.g. the quantity corresponding to point E.
- For the plant size at A, $SATC_1$ shows the cost of producing each output including Q_2. Hence SATC must lie above LAC at every point except A, the output level for which this plant size is best.

Short run and long run compared

- This argument can be repeated for other plant sizes. Hence $SATC_2$ and $SATC_3$, reflecting plant sizes at C and D, must lie above LAC except at points C and D themselves.
- In the long run the firm can vary all its factors and can generally produce a particular output more cheaply than in the short run, when it is stuck with the quantities of fixed factors it was using previously.
- A firm currently suffering losses because demand has fallen may make future profits once it has had time to build a plant more suitable to its new output.